CHAPTER SIX

ELASTICITY

Introduction:

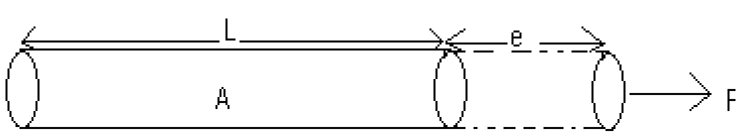
- The elasticity of a material is its tendency to regain its original shape or size, when the external or the deforming forces are removed from it.

Hook`s law:

- This states that provided the elastic limit is not exceeded, the extension of a material is directly proportional to the applied force or load.
- Another way of stating this law is that within the limit of elasticity, the stress is directly proportional to the strain.
- If S_S = Stress and S_n = strain, than $S_S \propto S_n$ => $S_S = E \times S_n$, where E is a constant known as the modulus of elasticity and E = $\frac{S_s}{S_n} = \frac{Stress}{Strain}$.

Young`s modulus of elasticity:

- This is the force which acts per unit area, which is capable of doubling the length of a material within the limit of elasticity.
- Its S.I unit is Nm^{-2} or the pascal (Pa).



- If a force or tension is applied to the end of a wire whose crossectional area is A, then the tensile stress = the force per unit area = $\frac{F}{A}$.
- If the extension of the wire is e and its original length is l, then the tensile strain = the extension per unit length = $\frac{e}{l}$.
- The modulus of elasticity of the material, which is called the Young`s modulus (E) is difined as E = $\frac{tensile\ stress}{tensile\ strain}$
- => E = $\frac{\frac{F}{A}}{\frac{e}{l}}$
- Even though strain has no units since it is a ratio of two lengths, stress has units such as Nm^{-2}.

(Q1) A 2kg mass is attached to the end of a vertical wire of length 2m and diameter 0.64cm, and the extension is 0.60mm. Calculate the Young`s modulus.

Soln:

Mass = m = 2kg.

Length = l = 2m.

Diametre = d = 0.64cm, => r = 0.32cm = $\frac{0.32}{100}$

= 0.0032m.

Extension = e = 0.60mm

$= \frac{0.60}{1000} = 0.0006$m.

Force = m $\times g = 2 \times 10 = 20N.$

Area of the wire = $\pi r^2 = 3.14 \times 0.0032^2 = 3.2 \times 10^{-5} m^2$

Tensile stress = $\frac{F}{A} = \frac{20}{3.2\times10^{-5}}$

$= \frac{20}{0.000032} = 625000Nm^{-2}$

- Tensile strain = $\frac{extension}{length}$

  $= \frac{0.00060}{2} = 0.00030$

  Young`s modulus = $\frac{tensile\ stress}{tensile\ strain}$

  $= \frac{625000}{0.00030} = 19.5 \times 10^8$

  N/B: Young`s modulus E, is calculated from the ratio $\frac{tensile\ stress}{tensile\ strain}$,

  $only\ when\ the\ wire\ is\ under\ elastic\ condition, i.e. when\ the\ elastic\ limit$

  $is\ not\ exceeded. N/B$:

- To convert a length given in millimetres into metres, we either multiply by 10^{-3} or divide by 1000.

- For example, 2mm = 2×10^{-3}m or 2mm = $\frac{2}{1000} = 0.002m.$

- To convert a length given in centimetres into metres, we either multiply by 10^{-2} or divide by 100.
- For example, a length of 8cm = 8×10^{-2}m, or 8cm = $\frac{8}{100} = 0.08m$.

Materials:

Strength: This is the resistance of a metal to stress when it is under a steady load.

- Its S.I unit is KN/m^2.

Tensile strength: This is the maximum load which a metal can sustain before breaking.

Plastic deformation: When an object can no longer regain its original size or shape after the removal of the deforming force, then it is said to have undergone plastic deformation.

Ductile substances: These are substances which lengthen considerably and undergo plastic deformation, until they break.

- Examples of such materials are iron and steel.

Brittle substances: These are substances which break just after the elastic limit is reached, and an example is glass.

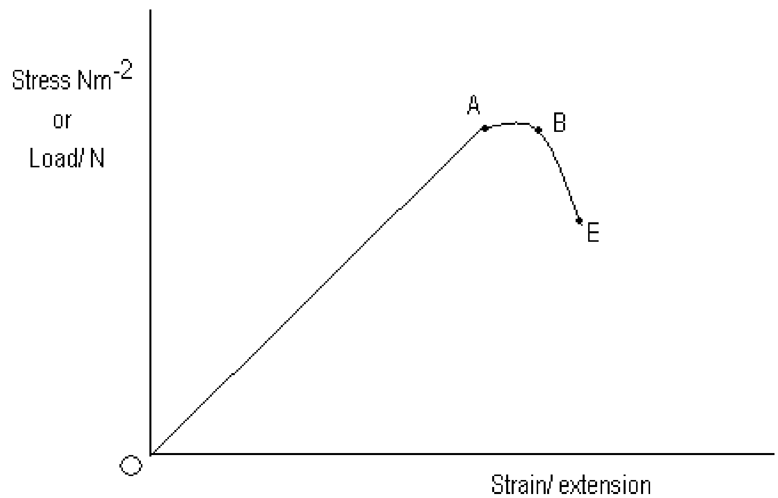
Permissible or safe working stress: This is the stress which a metal can withstand without any fear of it breaking.

Safety factor:

- This is the ratio of the tensile strength to the permissible tensile stress.
- Safety factor = $\frac{tensile\ strength}{permissible\ tensile\ stress}$
- In machines or structures, this permissible tensile stress is always far below the tensile strength and also below the stress which corresponds to the elastic limit.

Load/ extension curves:

(1) For a brittle substance:



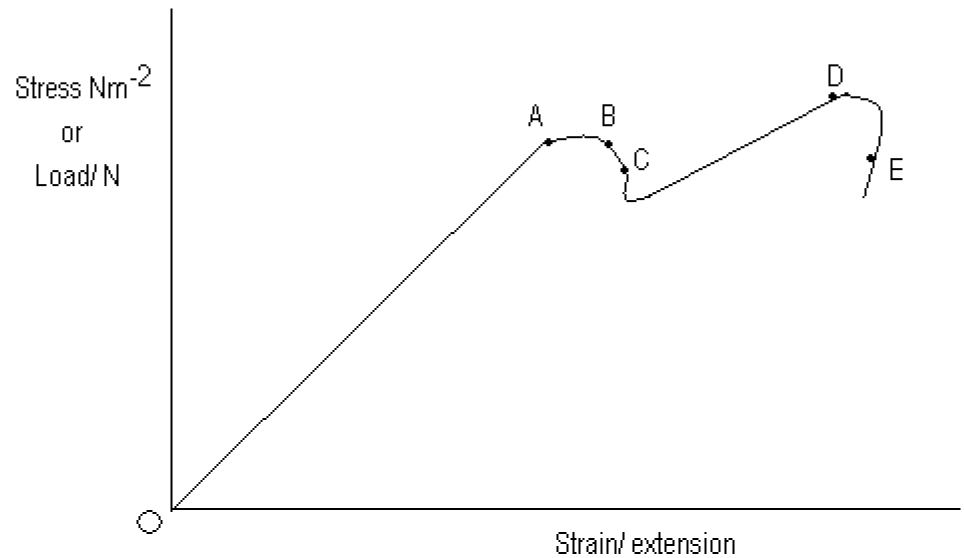
A = the point of proportionality.

B = the elastic limit or the point of elasticity.

E = the point of fracture (the material will break at this point).

OA = Elastic deformation.

(2) For a ductile material:



A = the point of proportionality.

B = the elastic limit.

C = the yield point i.e. the point on the load – extension diagram, at which permanent deformation is first observed.

D = the maximum (breaking) tensile stress point i.e. the point up to which forces can be applied, without unnecessarily extending or breaking the material.

E = The fracture point i.e. the point at which the material breaks.

OB = the elastic deformation stage.

CD = the plastic deformation stage.

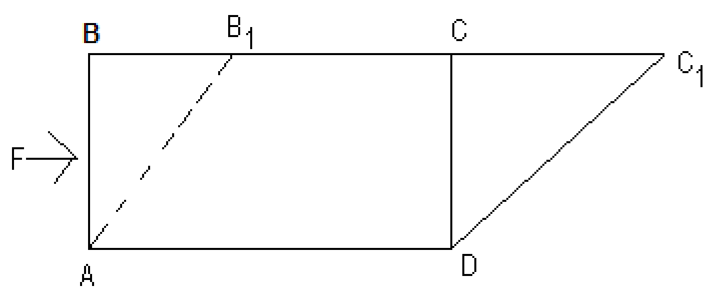
Compressive stress:

- When a material iis subjected to an increased pressure, the material contracts.

- The compressive force on the material is the increased in force per unit area.

Shear stress:

- This is the tangential force which acts per unit area on the free surface of the material.



- In this case, the tangential force tends to make the part on one side of the surface, slide past the part on the other side of the surface.

- In the given figure, AD is fixed and BC has slided to B_1C_1.